Mathematica a Matematické soutěže

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Proč právě matematické soutěže

Chceme vytáhnout elitu
Řešit úlohy, které vyžadují hlubší vhled do matematiky

- Co se změnilo

Řada matematických problémů se dá matematizovat (ref/ForAll)

```
ForAll[{a, b}, a > 0 && b > 0, (a + b) / 2 ≥ Sqrt[a b]]
Resolve[%]
```

$$\forall_{\{a,b\},a>0\&\&b>0} \frac{a+b}{2} \geq \sqrt{a\,b}$$

```
True
```

probability

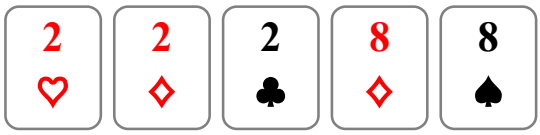
Input interpretation:

poker hand	type	full house

Description:

three matching cards of one rank and two matching cards of another rank

Example of a 5- card full house:



Properties: Show derivations

	number of possible hands	approximate probability	approximate chance
5- card hand	3744	0.001441	≈ 1 in 694
7- card hand	3 473 184	0.02596	≈ 1 in 39

(assuming random selection from a standard 52- card deck)

(the value of a 7- card hand is determined by its best 5- card subset)

Comparisons among 5- card poker hands:

	number	approximate probability	approximate chance
hands of lesser value	2 594 552	0.9983	≈ 1 in 1
hands of this type	3744	0.001441	≈ 1 in 694
hands of greater value	664	2.555×10^{-4}	≈ 1 in 3914

(assuming random selection from a standard 52- card deck)

WolframAlpha

Řada problémů se dá vypočítat hrubou silou

Určete poslední trojčíslí

$$\sum_{i=1}^{2014} i!$$

5 728 378 915 781 429 571 751 947 256 662 450 928 523 199 675 956 351 701 710 402 194 704 138 ∴
080 389 437 828 254 515 803 342 357 592 618 033 864 517 133 849 479 571 293 751 370 749 449 ∴
986 107 925 659 319 161 903 866 944 536 641 603 856 781 209 769 916 719 955 705 241 536 538 ∴

980 302 170 297 658 868 020 235 378 636 449 975 226 802 679 228 712 909 439 952 926 114 259 ∴
372 699 904 547 951 433 229 111 264 560 137 590 204 056 833 815 277 822 155 420 330 728 437 ∴
235 926 156 081 383 032 578 833 760 109 500 676 798 260 547 991 428 306 123 607 609 768 367 ∴
376 505 713 301 375 722 499 654 994 264 018 269 678 975 908 089 476 544 010 693 648 208 538 ∴
619 343 506 822 391 242 069 993 603 566 068 512 507 090 024 582 538 315 409 966 791 663 262 ∴
527 472 120 481 652 287 787 096 840 761 473 231 651 710 931 801 171 616 452 699 330 692 187 ∴
163 357 714 498 345 240 777 686 254 892 555 725 765 176 910 362 491 272 973 510 652 324 940 ∴
841 889 918 995 329 532 633 446 397 618 085 634 049 105 021 679 679 470 967 063 739 545 408 ∴
506 190 337 196 866 131 902 435 027 883 370 337 357 670 326 171 522 860 130 398 993 935 812 ∴
414 753 631 792 402 690 199 854 984 467 284 917 405 851 458 621 193 611 144 622 723 929 080 ∴
967 818 063 040 440 413 424 299 106 103 930 731 021 599 798 396 726 355 165 247 587 771 765 ∴
277 427 326 350 372 469 812 489 115 544 632 443 621 490 939 985 093 001 604 269 709 747 073 ∴
761 334 530 781 826 879 833 459 892 982 651 385 613 517 707 946 878 969 279 051 971 778 218 ∴
971 499 821 092 745 181 210 838 898 784 658 644 819 546 149 783 087 545 361 540 100 074 312 ∴
584 382 548 027 988 196 918 848 551 718 769 347 429 548 430 958 461 958 296 152 121 431 080 ∴
504 054 811 005 729 419 108 477 641 695 208 013 411 364 502 427 996 885 953 910 352 982 688 ∴
591 028 392 960 687 971 821 607 318 315 648 559 949 547 205 628 069 194 735 927 876 125 223 ∴
798 951 086 765 441 292 830 348 755 746 621 946 541 392 845 105 834 294 337 264 402 806 563 ∴
506 144 133 042 298 430 375 690 184 584 513 197 040 973 925 476 894 143 481 820 453 936 202 ∴
823 881 744 321 199 680 357 282 969 217 818 345 944 387 486 587 497 514 014 258 424 969 242 ∴
103 464 787 110 753 924 932 802 041 254 537 968 768 007 288 263 666 158 652 121 226 665 944 ∴
533 429 717 750 758 019 455 762 388 204 352 636 312 386 894 414 854 746 387 093 439 375 438 ∴
356 407 293 495 057 208 346 991 329 289 835 294 803 258 208 013 260 965 622 588 362 895 383 ∴
113 240 218 232 221 858 044 028 323 769 632 523 247 322 737 302 903 902 109 865 989 407 527 ∴
872 072 046 413 957 016 008 582 774 919 303 719 021 668 470 222 140 581 593 499 891 684 741 ∴
814 329 469 460 472 781 330 674 800 819 824 427 205 874 255 164 953 668 137 504 479 397 278 ∴
434 775 701 986 256 163 184 273 955 885 865 968 915 905 207 299 344 325 957 326 004 392 779 ∴
477 327 677 461 921 833 735 092 514 336 120 499 033 000 320 818 896 513 597 251 537 463 383 ∴
090 639 634 333 979 530 301 692 108 345 796 034 232 774 698 366 845 977 727 389 637 956 123 ∴
903 591 771 941 721 584 717 748 994 974 909 454 368 153 779 537 241 515 986 678 109 621 976 ∴
088 044 179 692 880 015 961 379 232 335 940 845 323 373 418 732 994 800 886 341 567 384 197 ∴
074 282 386 816 011 721 958 760 108 453 147 970 286 761 835 006 286 010 036 525 653 851 995 ∴
627 671 769 586 756 657 051 049 097 907 511 471 440 550 201 565 134 246 059 441 178 236 039 ∴
893 718 427 859 384 347 787 743 097 182 388 041 776 658 551 776 950 311 081 455 741 771 378 ∴
544 612 828 283 133 761 477 485 574 396 617 397 530 720 642 721 650 046 955 354 227 567 725 ∴
147 864 044 448 949 205 258 763 616 272 378 662 886 693 766 830 970 499 765 554 515 665 432 ∴
792 580 330 690 127 792 253 601 550 237 343 149 223 455 502 905 993 278 581 172 649 492 829 ∴
047 905 337 082 901 102 066 748 906 235 673 066 914 773 110 429 546 348 605 163 776 209 338 ∴
736 849 045 783 473 587 926 491 600 065 200 310 668 934 605 806 355 001 279 155 930 204 607 ∴
848 294 264 892 905 050 074 810 028 550 367 330 761 704 209 120 510 590 823 352 756 604 701 ∴
281 530 383 263 820 299 441 502 392 208 654 558 458 412 397 323 861 834 916 903 979 011 262 ∴
114 749 087 939 584 096 914 709 156 066 949 147 422 089 121 390 948 387 331 629 820 578 767 ∴
528 413 518 206 908 733 963 222 634 949 932 695 691 381 811 729 687 959 191 416 901 549 331 ∴
054 233 800 064 361 509 409 373 636 032 259 794 290 789 170 877 420 997 883 100 941 592 006 ∴
805 290 646 120 198 499 486 473 805 953 903 900 632 092 780 761 773 077 569 980 164 442 237 ∴
890 296 202 571 081 967 560 195 233 623 791 714 670 417 012 855 269 937 159 968 947 489 612 ∴
779 943 380 988 756 330 178 348 934 331 642 571 549 158 700 375 032 867 680 372 858 464 029 ∴
896 109 671 408 982 205 576 233 460 389 979 993 012 587 441 009 936 301 027 075 423 539 003 ∴
997 830 482 657 677 018 157 207 737 159 826 541 276 700 946 174 443 250 275 636 806 927 038 ∴
910 751 304 651 843 114 238 328 756 800 890 370 287 162 701 551 900 679 478 380 428 195 657 ∴
733 278 886 134 099 132 187 067 076 911 852 668 630 411 971 211 438 775 210 661 217 836 009 ∴
442 875 367 480 507 628 095 824 523 286 534 052 616 573 320 476 977 042 449 976 352 478 504 ∴
207 835 866 838 352 893 096 839 889 426 527 937 074 831 091 601 137 687 822 317 144 790 945 ∴
877 591 584 426 609 124 066 213 756 454 031 276 387 538 898 070 165 107 816 784 791 053 771 ∴
814 932 740 425 308 564 535 345 556 412 005 219 683 610 566 706 650 764 161 251 582 670 608 ∴
072 968 051 171 491 304 749 373 630 740 664 161 254 253 153 411 846 125 385 647 003 893 824 ∴

367 458 027 474 418 482 014 777 421 524 248 442 063 242 534 641 266 736 754 523 342 607 280 ↘
801 637 853 510 518 265 685 896 012 347 152 430 162 053 564 193 583 657 030 663 662 299 465 ↘
376 663 220 530 117 348 739 505 244 424 099 149 166 161 546 621 920 543 487 914 949 981 846 ↘
190 513 371 132 249 303 741 151 093 759 753 389 381 369 121 446 875 389 568 150 748 206 682 ↘
238 168 890 284 743 600 406 921 916 852 892 653 052 709 685 511 164 966 795 968 701 229 314 ↘
021 106 705 382 194 320 492 508 805 364 235 546 854 061 910 237 881 742 378 056 590 804 765 ↘
615 152 895 741 684 388 604 285 036 257 623 683 375 561 846 654 679 272 660 247 438 997 355 ↘
562 371 119 025 204 903 783 836 452 455 401 071 595 705 952 857 054 900 620 396 437 426 730 ↘
249 473 065 968 205 012 926 025 103 741 335 793 990 586 247 029 841 201 251 893 908 069 179 ↘
103 253 144 195 342 996 972 999 975 220 543 393 356 184 323 530 849 928 920 219 560 827 827 ↘
866 885 220 636 086 965 751 995 966 323 797 976 719 683 475 739 430 543 823 894 025 087 225 ↘
941 643 928 069 131 925 674 635 577 696 399 333 554 285 104 724 562 399 364 825 483 467 964 ↘
468 896 514 992 076 666 125 523 776 120 556 740 359 935 287 171 487 550 933 881 790 163 149 ↘
066 176 236 295 211 889 258 306 382 126 036 014 007 469 871 567 372 794 276 191 972 144 307 ↘
937 452 416 993 688 172 296 350 923 457 851 069 371 375 204 445 861 550 760 799 157 616 919 ↘
186 157 381 400 409 613 770 677 585 290 810 876 562 703 808 984 290 284 494 775 527 542 334 ↘
114 112 701 703 544 375 016 014 600 417 131 807 933 183 618 635 491 238 193 345 018 349 565 ↘
879 628 173 820 633 759 757 237 164 062 846 293 995 746 925 171 844 037 841 823 522 441 611 ↘
867 345 433 208 379 741 347 009 406 659 919 379 208 878 761 799 827 677 548 449 306 681 397 ↘
552 694 707 427 117 631 406 144 007 450 882 969 740 049 818 323 455 858 551 317 207 197 201 ↘
589 704 858 224 388 374 914 095 841 726 938 767 482 243 085 025 897 244 690 463 595 978 336 ↘
569 707 778 625 455 613 570 273 121 541 638 995 145 894 561 079 999 213 500 076 101 248 781 ↘
500 642 979 091 198 044 736 325 729 107 144 579 768 273 776 743 716 439 557 581 557 253 878 ↘
204 647 391 176 582 851 167 312 182 026 188 795 156 620 056 856 503 340 092 247 479 478 684 ↘
738 621 107 994 804 323 593 105 039 052 556 442 336 528 920 420 940 313

Slovo Mathematica

Hrubá síla

```
"Mathematica"
```

```
Characters["Mathematica"]
```

```
{M, a, t, h, e, m, a, t, i, c, a}
```

```
Subsets[Characters["Mathematica"], {3}]
```

```
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 {e, c, a}, {m, a, t}, {m, a, i}, {m, a, c}, {m, a, a}, {m, t, i}, {m, t, c},
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 {a, i, c}, {a, i, a}, {a, c, a}, {t, i, c}, {t, i, a}, {t, c, a}, {i, c, a}}
```

```
Subsets[Characters["Mathematica"], {3}] // Length
```

```
165
```

Kombinatorika

```
Characters["Mathematica"] // Length
```

```
11
```

```
Binomial[11, 3]
```

```
165
```

Co s tím

- Můžeme Wolfram Mathematicu a WolframAlpha zakázat

  ... ale to asi není cesta

- Můžeme nastavit úlohy tak, aby byly obtížné i pro počítač (případně uživatele)

Mathrace

http://brkos.math.muni.cz/mathrace/reseni2013.pdf

Uloha 7

```
Solve[a^6 - b^6 == 3367 && a > 0 && b > 0, {a, b}, Integers]
a * b /. %
```

```
{{a → 4, b → 3}}
```

```
{12}
```

Uloha 39

```
f[x_] := 2 x - 1
```

```
$RecursionLimit = Infinity
```

```
∞
```

```
Nest[f, x, 2]
```

```
-1 + 2 (-1 + 2 x)
```

```
Solve[Nest[f, x, 2014] == 2^2016 + 1, x]
```

```
{{x → 5}}
```

Matematická olympiáda

http://mo.webcentrum.muni.cz/media/1310159/abc64dom.pdf

Kategorie B úloha 1

```
Solve[{Abs[x - 5] + Abs[y - 9] == 6, Abs[x^2 - 9] + Abs[y^2 - 5] == 52}, {x, y}, Reals]
```

```
{{x → 1, y → 7}, {x → 1 + 4 √2 , y → -1 + 4 √2 }}
```

Kategorie C úloha 6

```
Select[Table[{i, IntegerDigits@IntegerPart[FractionalPart[N[√i]] * 100]},
  {i, 10 000}], #[[2]] == {9, 9} &]
```

```
{{2600, {9, 9}}, {2703, {9, 9}}, {2808, {9, 9}}, {2915, {9, 9}}, {3024, {9, 9}},
 {3135, {9, 9}}, {3248, {9, 9}}, {3363, {9, 9}}, {3480, {9, 9}}, {3599, {9, 9}},
 {3720, {9, 9}}, {3843, {9, 9}}, {3968, {9, 9}}, {4095, {9, 9}}, {4224, {9, 9}},
 {4355, {9, 9}}, {4488, {9, 9}}, {4623, {9, 9}}, {4760, {9, 9}}, {4899, {9, 9}},
 {5040, {9, 9}}, {5183, {9, 9}}, {5328, {9, 9}}, {5475, {9, 9}}, {5624, {9, 9}},
 {5775, {9, 9}}, {5928, {9, 9}}, {6083, {9, 9}}, {6240, {9, 9}}, {6399, {9, 9}},
 {6560, {9, 9}}, {6723, {9, 9}}, {6888, {9, 9}}, {7055, {9, 9}}, {7224, {9, 9}},
 {7395, {9, 9}}, {7568, {9, 9}}, {7743, {9, 9}}, {7920, {9, 9}}, {8099, {9, 9}},
 {8280, {9, 9}}, {8463, {9, 9}}, {8648, {9, 9}}, {8835, {9, 9}}, {9024, {9, 9}},
 {9215, {9, 9}}, {9408, {9, 9}}, {9603, {9, 9}}, {9800, {9, 9}}, {9999, {9, 9}}}
```

```
N[√2600, 15]
```

```
50.9901951359278
```

```
50.99^2
```

```
2599.98
```

```
Table[(i + 0.99)^2, {i, 60}]
```

```
{3.9601, 8.9401, 15.9201, 24.9001, 35.8801, 48.8601, 63.8401, 80.8201, 99.8001,
 120.78, 143.76, 168.74, 195.72, 224.7, 255.68, 288.66, 323.64, 360.62,
 399.6, 440.58, 483.56, 528.54, 575.52, 624.5, 675.48, 728.46, 783.44,
 840.42, 899.4, 960.38, 1023.36, 1088.34, 1155.32, 1224.3, 1295.28, 1368.26,
 1443.24, 1520.22, 1599.2, 1680.18, 1763.16, 1848.14, 1935.12, 2024.1,
 2115.08, 2208.06, 2303.04, 2400.02, 2499., 2599.98, 2702.96, 2807.94,
 2914.92, 3023.9, 3134.88, 3247.86, 3362.84, 3479.82, 3598.8, 3719.78}
```

Užitečné funkce

Výpis všech dělitelů zadaného čísla

```
Divisors[1 566 486]
```

```
{1, 2, 3, 6, 9, 18, 27, 54, 29 009, 58 018,
 87 027, 174 054, 261 081, 522 162, 783 243, 1 566 486}
```

Největší společný dělitel a nejmenší společný násobek

```
GCD[24 646, 21 546]
```

```
2
```

```
LCM[265, 21 546]
```

```
5 709 690
```

Převedení čísla na jeho cifry, jejich součet a součin

```
IntegerDigits[1 566 486]
Plus @@ %
Times @@ %%
```

```
{1, 5, 6, 6, 4, 8, 6}
```

```
36
```

```
34 560
```

Výpočet na zvolený počet cifer

```
N[π, 50]
```

```
3.1415926535897932384626433832795028841971693993751
```

Poděkování

Rád bych na tomto místě poděkoval za možnost realizovat projekt eVIK - výuka, individualizace, koučing, CZ 1.07/1.1.32/02.0132, který na naší škole - Dvořákovu gymnáziu a SOŠE, Kralupy nad Vltavou, probíhá v letech 2013 a 2014. Tento projekt je financován Evropským sociálním fondem a rozpočtem České republiky.

Dále bych rád poděkoval i projektu SVV 260098 - Studentský výzkum v oblasti didaktiky fyziky a matematického a počítačového modelování, za možnost hlouběji prohlédnout do zadané problematiky.